

مسئله ۱

Quiz

A) Prove that $\frac{d}{dz} (z^2 \bar{z})$ doesn't exist anywhere

B) show that $\frac{d}{dz} (\ln(z)) = \frac{1}{z}$

Complex integration

$$\int_a^b f(z) dz = \int_c f(z) dz \rightarrow \text{مسئله ۲}$$

$$\oint_c f(z) dz \rightarrow \text{مسئله ۳}$$

$$\boxed{1} \int_c f(z) dz = ?$$

$$f(z) = u + iv \quad ; \quad z = x + iy$$

$$dz = dx$$

$$dz = dx + i dy$$

$$\therefore \int_c f(z) dz = \int_c (u + iv)(dx + i dy)$$

$$= \int (u dx - v dy) + i(v dx + u dy)$$

1 Sec 7

(۱۰) تم نچلہ کلہ (۳) x، dx، dy، y اور دربارہ صفریہ

① Circle $(x - x_0)^2 + (y - y_0)^2 = a^2$

$$x = x_0 + a \cos t \quad ; \quad y = y_0 + a \sin t$$

$$dx = -a \sin t dt \quad ; \quad dy = a \cos t dt$$

② قلع ناقص

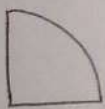
$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$$

$$X = X_0 + a \cos t$$

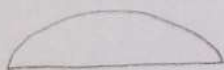
$$y = y_0 + b \sin t$$

$$dx = -a \sin t \, dt$$

$$dy = b \cos t \, dt$$



$$0 \leq t \leq \frac{\pi}{2}$$



$$0 \leq t \leq \pi$$



الفترات ←

$$0 \leq t \leq 2\pi$$

قطع زائد [3]

$$\frac{(x-x_0)^2}{a^2} - \frac{(y-y_0)^2}{b^2} = 1$$

$$X = X_0 + a \cosh t$$

$$y = y_0 + b \sinh t$$

$$dx = a \sinh(t) dt$$

$$dy = b \cosh(t) dt$$

2 sec 7

④ قطع مكافئ

④

$$y = ax^2$$

$$x = t$$

$$y = at^2$$

$$dx = dt$$

$$dy = 2at dt$$

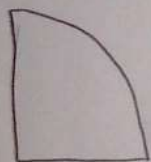
⑤ $|z - z_0| = a$

$$x = x_0 + a \cos \theta$$

$$y = y_0 + a \sin \theta$$

$$dx = -a \sin(\theta) d\theta$$

$$dy = a \cos \theta d\theta$$



$$0 \leq \theta \leq \frac{\pi}{2}$$



$$0 \leq \theta \leq \pi$$



$$0 \leq \theta \leq 2\pi$$

□ Evaluate $\int f(z) dz$ From $0+i$ to $2+5i$

where $f(z) = (3x+y) + i(x-2y)$

a) along $y = x^2 + 1$

b) along line $(0,1)$ to $(0,5)$

solution

$$f(z) = (3x+y) + i(x-2y) \quad , \quad dz = dx + i dy$$

$$a) \quad y = x^2 + 1$$

$$f(z) = (3x + x^2 + 1) + i(x - 2x^2 - 2)$$

$$dz = dx + i dy$$

$$= dx + i 2x dx$$

$$\therefore \int_C f(z) dz = \int_0^2 \left[(3x + x^2 + 1) + i(x - 2x^2 - 2) \right] (dx + i 2x dx)$$

$$= \int_0^2 (3x + x^2 + 1) dx + i(x - 2x^2 - 2) dx +$$

$$i(3x + x^2 + 1) 2x dx - (x - 2x^2 - 2) 2x dx$$

$$= \int_0^2 (3x + x^2 + 1 - 2x^2 + 4x^3 + 4x) dx$$

$$+ i(x - 2x^2 - 2 + 6x^2 + 2x^3) dx$$

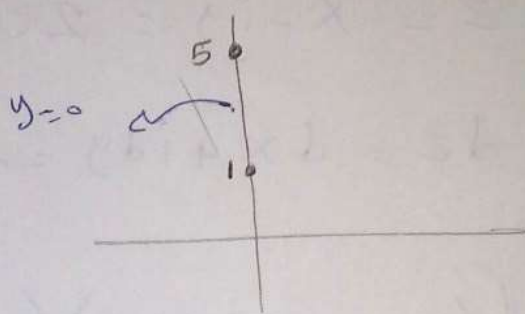
$$= \int_0^2 (7x - x^2 + 1 + 4x^3) dx + i(3x + 4x^2 - 2 - 2x^3) dx$$

$$= \left(\frac{7x^2}{2} - \frac{x^3}{3} + x + x^4 \right) \Big|_0^2 + i \left(\frac{3x^2}{2} + \frac{4x^3}{3} - 2x + \frac{x^4}{2} \right) \Big|_0^2$$

$$= \frac{88}{3} + i \frac{62}{3}$$

b)

~~$x=0$~~ $x=0 \quad dx=0$



$$f(z) = y - izy$$

$$dz = i dy$$

$$\int_C f(z) dz = \int_1^5 (y - izy) i dy$$

$$= \int_1^5 i y dy + 2y dy$$

$$= i \frac{y^2}{2} \Big|_1^5 + y^2 \Big|_1^5$$

$$\int_C f(z) dz = 24 + i12$$

* evaluate $\int \bar{z} dz$ for $x = 2\cos t, y = \sin t$

From $t = 0$ to $t = \frac{\pi}{2}$

$$\bar{z} = x - iy = 2\cos t - i\sin t$$

$$dz = dx + i dy = -2\sin(t)dt + i\cos(t)dt$$

$$\int_0^{\frac{\pi}{2}} (2\cos t - i\sin t) (-2\sin(t)dt + i\cos(t)dt)$$

$$= \int_0^{\frac{\pi}{2}} -4\cos t \sin t dt + i2\sin^2 t dt + i2\cos^2 t dt + \sin t \cos t dt$$

$$= \int_0^{\frac{\pi}{2}} -3\cos t \sin t dt + i2 dt$$

$$= \int_0^{\frac{\pi}{2}} -3 \frac{\sin 2t}{2} + 2i dt = \frac{3}{4} \cos 2t + i2t \Big|_0^{\frac{\pi}{2}}$$

$$= \frac{-3}{4} + \pi i$$

7] تكامل على مسار مغلق

$$a) \oint_C f(z) dz = 0$$

← إذا كانت الدالة $f(z)$ ليس لها مقام ولا تحتوي على \bar{z} أو $|z|$ أو $\arg z$ مقام لا تقع داخل المنحنى (analytic)

$$b) \oint_C \frac{f(z)}{z-a} dz = 2\pi i f(a)$$

$$c) \oint_C \frac{f(z)}{(z-a)^{n+1}} dz = \frac{2\pi i}{n!} \left. \frac{d^n f(z)}{dz^n} \right|_{z=a}$$

← إذا كانت الدالة تحتوي على مقام واحد وصفره داخل المنحنى C .

الأذكاري

(a) تعويض مباشر.

(b) فيه أكثر من قوس تحت واحد فيهم يقع داخل المنحنى والباقي لا.

(c) أكثر من قوس تحت واحد أو مقامه تقع داخل المنحنى.

$$\oint_C = \oint_{C_1} + \oint_{C_2}$$

Evaluate

$$\boxed{1} \oint_C e^{10z} dz$$

$$b) \oint_C \frac{z^2}{z-3} dz \quad |z|=1$$

$$\boxed{c} \oint_C \operatorname{sech}(z) dz$$

$$\boxed{d} \oint_C z \cdot e^{-z} dz$$

$$\boxed{e} \oint_C \frac{1}{z^2 + 2z + 1} dz$$

$$\boxed{f} \oint_C \frac{2z-1}{z^2 - z} dz$$

$$|z| = \frac{1}{2}$$

Sol

$$\boxed{1} \oint_C e^{10z} dz = 0$$

$$\boxed{2} \oint_C \frac{z^2}{z-3} dz = 0$$

$$|z|=1$$



$$\boxed{3} \oint_C \operatorname{sech}(z) dz = 0$$

$$\cosh z = \frac{e^z + e^{-z}}{2}$$

$$\oint_C \operatorname{sech}(z) dz = \frac{2}{e^z - e^{-z}} = 0$$

$$\boxed{8} \sec 7$$

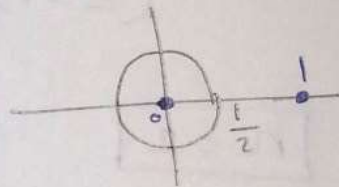
$$\boxed{4} \int_C z \cdot e^{-z} dz = 0$$

$$\boxed{5} \int \frac{1}{z^2 + 2z + 1} dz = 0$$

$$|z| = 1 ; z = -1 \pm i$$

$$\boxed{6} \oint \frac{2z-1}{z^2-z} dz = \oint \frac{2z-1}{z(z-1)} dz$$

$$|z| = \frac{1}{2}$$



$$= \oint \frac{\boxed{\frac{2z-1}{z-1}} \rightarrow f(z)}{z} = 2\pi i f(a)$$

$$= 2\pi i f(0) = \boxed{2\pi i}$$

$$\boxed{7} \oint_C \frac{7z-6}{z^2-2z} ; |z| = 6$$

$$= \oint \frac{7z-6}{z(z-2)}$$

النقطتين داخل المنحنى .

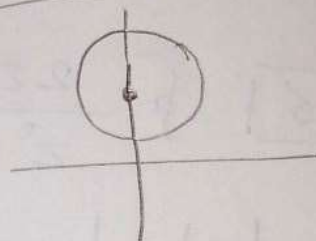
$$= \int_{C_1} \frac{\frac{7z-6}{z-2}}{z} + \int_{C_2} \frac{\frac{7z-6}{z}}{z-2}$$

$$= 2\pi i f(0) + 2\pi i f(2)$$

$$= 6\pi i + 8\pi i = 14\pi i$$

$$\boxed{8} \int \frac{z^2 e^{2z^2}}{(z-i)^3}$$

$$|z-i|=1$$



$$\boxed{n=2}$$

$$\text{Cause } n+1=3$$

$$\oint = \frac{2\pi i}{2!} f(z) = \frac{2\pi i}{2!} e^{2z^2}$$

$$f'(z) = 4z e^{2z^2}$$

$$f''(z) = 4 e^{2z^2} + 16z^2 e^{2z^2}$$

$$f''(i) = 4 e^{-2} + 16(-1) e^{-2} = -12 e^{-2}$$

$$\oint = \frac{2\pi i}{2!}$$

$$\oint = \frac{2\pi i}{2!} (-12 e^{-2}) = -12 e^{-2} \pi i$$

$$\boxed{10} \text{ sec 7}$$